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ABSTRACT

Presented is an overview of some specific schemes that have been used successfully by teachers throughout New York State to strengthen basic mathematics skills. Components offer ideas that have been successful with primary, intermediate, and secondary students. The contents of this English language edition are identical to the other foreign language editions. In addition to the Foreword, there are sections on: (1) Some Brief Observations About Strengthening Mathematics Skills; (2) The Balanced Mathematics Program; (3) "Par"--Puzzles+Arithmetic=Remediation; (4) Regrouping in Subtraction; (5) Money Games; (6) A Visual Sequence for Teaching Fractions; (7) A Space to Carry in Simple Addition and Multiplication Examples; (8) Grid Paper Computation; (9) The Need for Math Reading Skills; (10) A Structural Approach to Multiplication; (11) The Electronic Calculator in Remedial Mathematics; (12) Nature's Mathematics; and (13) Additional Teacher Designed Ideas. (MP)

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MATHEMATICS

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IDEAS FOR STRENGTHENING **Mathematics** SKILLS

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SE 035 820

MATHEMATICS

IDEAS FOR STRENGTHENING **Mathematics** SKILLS

The University of the State of New York
THE STATE EDUCATION DEPARTMENT
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FOREWORD

With all the publicity lately about the "return to the basics," it is not surprising that school personnel are looking for ideas to strengthen the skills portion of their overall mathematics course of study. These materials do not claim to provide teachers with in-depth diagnostic instruments or a hocus pocus, fool-proof panacea. The purpose of this publication, however, is to give an overview of some specific schemes that have been used successfully by teachers throughout New York State to strengthen mathematics skills. The various components of this publication offer ideas that have been successful with primary, intermediate, and secondary students. The reader is encouraged to review all components with the understanding that each may suggest an idea that, if not appropriate as written, may be modified to fit their situation.

This publication is a joint effort of the Bureaus of Mathematics Education and Bilingual Education. It was developed under funds provided by Title VII of the Elementary and Secondary Education Act of 1965. However, the opinions expressed herein do not necessarily reflect the positions or policy of the U.S. Office of Education. Materials were solicited from various mathematics educators throughout New York State by Lynn A. Richbart, Associate in Mathematics Education, and Louise Lutz, Title I Mathematics Coordinator for the city of Syracuse. Dr. Lutz developed the original manuscript which was then edited by Lynn A. Richbart and Aaron L. Buchman, Associates in Mathematics Education. The final editing and preparation of the manuscript for printing was done in the Bureau of General Education Curriculum Development.

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In addition to this English version this publication will also soon be available in Creole, Greek, Italian, and Spanish.

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Some Brief Observations About Strengthening Mathematics Skills

The purpose of this article is to give an overview of some specific approaches which are valuable in strengthening mathematics skills.

Manipulative Materials

The use of manipulative materials and the laboratory approach in general means different things to different people. Here we mean the use of a wide variety of concrete things. Some may be commercially slick plastic while others may be less sophisticated homemade materials. Surprisingly, various studies have shown that the latter type of materials can be more appealing to the student and, perhaps not so surprisingly, are used more often by the teacher.

Many teachers are now familiar with books by Edith Biggs, the Nuffield Project, N.C.T.M. Experiences in Mathematics Ideas, the numerous articles in the Arithmetic and Mathematics Teacher, and N.Y. State Publications.* Suffice it to say that the stress is placed on gradually moving from the concrete to the abstract. The atmosphere, although seemingly free, is usually quite structured. The teacher must know what materials are appropriate for a particular concept and must keep careful records on what the student has accomplished. In many situations the students are also required to keep a log of their experiences.

Alternate Algorithms

Although the jury is still out, there appears to be some value in showing students that computational processes may be performed using different rules or algorithms. In fact, most elementary texts develop certain algorithms such as multiplication and division through a series of refinements until the more efficient rule is reached. As an example, notice this development:

```
75 ⌈2550⌋
    750   75 x 10
   1800
    750   75 x 10
   1050
    750   75 x 10
    300
    225   75 x  3
     75
     75   75 x  1
      0        34
```

```
75 ⌈2550⌋
   2250   75 x 30
    300
    300   75 x  4
      0        34
```

```
        34
75 ⌈2550⌋
   2250
    300
    300
      0
```

Although each example in this development is based on the understanding of the previous example, some students seem to get lost somewhere in the middle. To them, the final form has no relationship to the intermediate stages.

Suggestions for teaching mathematics using laboratory approaches.

Textbooks seem to abound with examples of this type of "stage development" which eventually leads to a standard traditional algorithm. (In this case, the long division algorithm.)

There are many other alternative algorithms, not ordinarily found in textbooks, which tend to provide more stimulation to the students to practice their computational skills.*

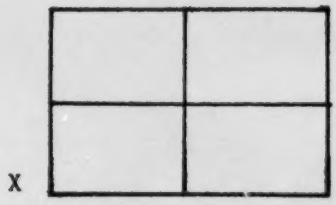
Games

Very few games exist that don't use some form of mathematics. Whether it be keeping score, making change, or merely moving a certain number of spaces, games are a natural asset to computational practice. Many teachers use games as a reward or during those last few minutes of pre-holiday class sessions.

As with manipulative materials, there are plenty of games from which to choose. There are those commercial games with specific mathematics objectives or embedded mathematics requirements. There are teacher-made games either developed from the many books of mathematical recreations or, best of all, from the teacher's or student's creative imagination.

Since games have at least this dual role of recreational enjoyment and computational skill-building, the teacher should be clear as to the purpose desired at a given time. If the purpose is skill-building, the teacher should know which skills are being reinforced by a particular game and make a decision as to whether or not the players should both have the same mathematical proficiencies.

Games, such as the one described here, may combine several mathematical ideas. For example, a simple game is to have each student make a box with four compartments such as this:



The teacher or game leader then selects four digits at random by either drawing them out of a hat, spinning a spinner or rolling a die. As the numbers are chosen, the students fill in the boxes in any order they wish. Once the four digits are picked they perform a desired operation such as in this example, multiplication. Those with the highest score win.

Such a simple game involved practice with a computational skill, understanding of the importance of place value to win the game and even some intuitive feeling for the probability that certain digits might be called.

*Several of these are illustrated in the article, "PAR" - Puzzle + Arithmetic = Remediation, included in this publication.

Relevant Applications

Oftentimes, we develop an application for computational practice which we think is relevant to the student. Yet, many times it is not. Income Tax, insurance premiums, and home mortgages may be important topics to cover, but for many students their current relevance is minimal. If you have an older general math student filing for a tax refund, then you have relevance.

One avenue of attack is to find out what interests your students. What are their hobbies? Do they like sports? If so, which ones? Are they expected to help around their home with specific chores or do they help their parents or older brothers and sisters with more complex jobs?

Once you really get to know your students, relevant applications become obvious. One student might be very interested in baseball. Sports in general are just full of statistics and therefore computational practice. For example, the batting average* as shown below is a simple division of the number of base hits (H) by the number of times officially at bat (AB). The "official" eliminates walks, sacrifices, or times hit by pitch.

AB = 524 H = 154

Average .294

$$524\overline{)154.0000} = .2938$$

The batting average is rounded off to the nearest thousandth and the decimal point is dropped - i.e., in the above example, we would say that the batting average is 294.

Diagnosis and Computation

When students think of mathematics class, they usually think of daily homework and numerous tests. Both of these can be looked upon either as a lot of unpleasant paper work or as valuable input for diagnosis. If the magnitude of the task of looking at each homework exercise appears frightening, don't reject the whole idea. Careful diagnosis should be done immediately following a computational skill presentation in class and thus, the number of exercises assigned should be quite limited. You might assign more numerous examples later to sharpen skills, but even at that point, a careful diagnosis of specific students or particular exercises may do much to save you work in the long run. Another point to mention on homework is to encourage your students to show most of their thinking on the paper so that you may have a clue as to what happened if their final answer is wrong.

This last point is important on written class tests as well. If the test is being used to help rather than just rate the student, it is important to see all of a student's input. You may find it desirable to test some students orally. Have them tell you what they are thinking as they perform a given operation. For example, if they don't remember 7 x 8 = 56, how do they try to arrive at the answer with what they do remember?

*Taken from N.Y. State Publication, *Arithmetic Around the Home*, available in English and Spanish.

Teaching computation can be a frustrating experience. The fact must be accepted that many students will make frequent mistakes. Instead of dwelling on a student's deficiencies, build on his proficiencies. Some may call it "success orientation" or "no-failure teaching," but the point is that building a student's confidence can be the key in teaching something like computation which is based on a hierarchy of skills.

The Balanced Mathematics Program

For some, mathematics has been one of the most disliked and boring subjects taught in schools. Most students who disliked mathematics usually were low achievers in that subject. Mathematics for years seemed to be page after page of practice work followed by corrections the following day. Besides creating a hatred for mathematics, we were also graduating a generation who couldn't do simple mathematics computation required in everyday life.

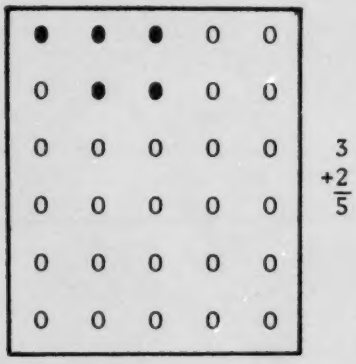
To try to correct these deficiencies, remedial mathematics programs were initiated in nearly every school district in our state. But nearly all of these programs were designed with a very narrow view of mathematics and with specific limitations relating to the abilities and needs of remedial mathematics students. It is for these reasons and others that a new approach in the teaching of mathematics was needed. An approach developed within the Niagara Falls School System's ESEA Title I program has been labeled the "Balanced" or "Total Mathematics Program."

In this balanced approach there are three equally important parts; *teaching*, *reinforcement*, and *application*. Certainly these are not new terms to most educators. But in our definitions of these terms and their relationships to one another, Niagara Falls hopes a new approach in dealing with mathematics will be visible. To achieve this goal we have picked one area of the mathematics curriculum, *addition*, to serve as a model.

$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ approach

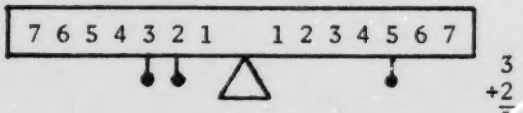
Ideally, a student would spend one-third of his time in dealing with mathematics being taught by a teacher. There is no substitute for *direct teaching* in many areas of the math curriculum. The teacher should make use of a variety of manipulative materials such as blocks, counting sticks, pegboards, abacus, base ten blocks, and other home made devices with an emphasis on student exploration, manipulation, and concept development. Children should be allowed to develop through the natural stages of learning from the concrete, to the pictorial, to the abstract. Examples are given on the following page.

Concrete illustrations to the teaching of addition abstractions.



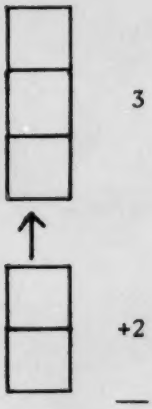
pegboard

3
+2
5



balance

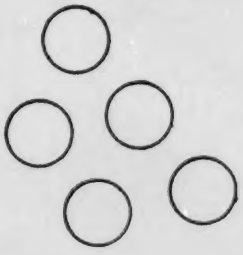
3
+2
5



3

+2

5



counting disks

3
+2
5

The second area in the balanced program is *reinforcement*. Again one-third of a child's time with mathematics should involve practice. We sometimes forget that everyone needs to practice - even the child that shows a mastery of a certain skill. We know from our own lives that if we cease to practice a certain skill, we soon lose some of our ability in that area. The same is true of a child's work in mathematics.

In the past, reinforcement in the classroom meant two things: dittos and workbooks. Today, a teacher has a tremendous variety of materials and machines to use. A partial listing includes calculators, computers, filmstrips, sound tapes, skill kits, teaching machines, as well as teacher-designed games and activities. We feel that teacher-made games and activities offer the most possibilities for individualization as well as being highly motivational. Studies have shown that a child's attitude towards mathematics and his self-concept of his ability in mathematics affect his success in mathematics.

Three important points should be kept in mind when using games and activities for reinforcement with children:

- *You* design the game for the child after you have diagnosed a need or the child has shown an interest.
- Games and activities do occasionally teach, but if you bet on them to do so consistently, you'll lose. There's no substitute for direct teaching.
- Whenever possible, games and activities should incorporate the use of the various manipulatives used previously in teaching.

Let's take a look again at some of the possibilities in addition.

Activity 1

PLAY BALL! 3 + 4

Why designed

To help a child whose interest was baseball, practice number facts.

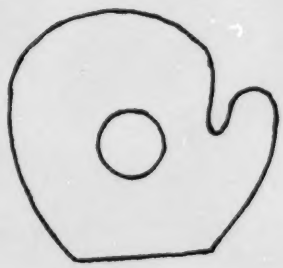
How

A flashcard is shown. Child tries to "catch" the answer with hole in glove.

Ballfield

2	9	11	14	4
7	18	5	3	16
1	8	6	10	18
12	15	19	13	20

Glove



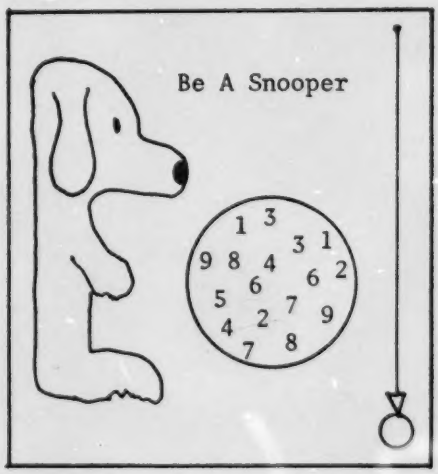
Activity 2

Why designed

To help children see the relationship between addition and subtraction.

Children use the magnifying glass to find number families and record their work - eg.,

3,4,7 $3 + 4 = 7$ $7 - 3 = 4$
$4 + 3 = 7$ $7 - 4 = 3$

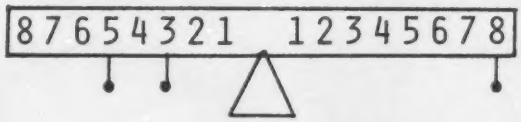


magnifying glass

Activity 3

Why designed

To help a child begin to learn his number facts and also to encourage problem solving.



5 + 3 = 8
_ + _ = 8
_ + _ + _ = 8
ETC.

record sheet

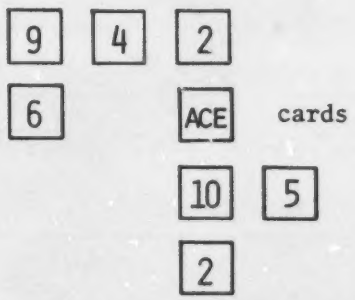
How many ways can we name 8 using 2 weights? 3 weights? 4 weights?

Activity 4

Number Scrabble

Why designed

To strengthen number fact recognition.



Need - 1 deck of cards
Rules - deal 7 cards as if they they were regular scrabble tiles
Object - make 15

Finally, we have the *application* area. Here we try to show children that mathematics isn't an island separated from all other content areas or our everyday lives. Children need to be made aware of and involved with mathematics in the "real world." This is the part of the total mathematics program in which we too often expect a child to sit down, do something, then hand it in every 30 minutes. Students, even poor ones, need the opportunity to learn how to deal with many-faceted problems that cannot be solved in 30 minutes. They need to face problems that require them to use various mathematical tools, that incorporate science, reading, language arts, etc. They need to be involved in problem-solving situations that make them responsible for decision making, record keeping, and reporting outcomes of their research. They should have the opportunity to work in groups towards a common goal. The possibilities in this area are unlimited and should require a student to use what he has previously learned. The interests of the student should be a major consideration.

The following represents one possibility.

I. Topic: Sports

II. Theme (objective)

This unit is designed to correlate track and field events with various concepts in mathematics.

III. Behavioral Objectives

A. to build confidence in children

B. to associate sports with mathematics

C. to provide actual practice in the various mathematics concepts

D. to reinforce understanding of mathematics concepts

E. to show the use of various materials to compute data

F. to provide experience in the computational area

G. to teach children to listen and follow directions

H. to create enthusiasm in the children

IV. Procedure (planning)

A. Introduction (set up) - create interest by talking about what is going to take place during the summer. Continue by setting up the classroom. Have the children make various sports scenes around the room. Set up sports displays. This particular part of the unit will involve art work which should be made interesting for the children.

B. When the environment of the class is set, go outside and check up on the surrounding area. See what is available to help you conduct track and field events.

C. At this time, start the various events, one event at a time or various events each day. This would be optional.

D. Events

1. football kicking
   applications - measurement, average, graphing

2. ball throwing - basketball, football
   applications - graphing, measurement, percent, average

3. jumping - indoor, outdoor
   applications - measuring length

4. running - relay, distance, sprints
   applications - measuring time

5. obstacle course
   applications - timing, angles, jumping measurement

Just about every event will require graphing which will involve considerable computation. Questions such as, how much?, how many?, how fast?, how predictable?, how does?, why are?, what is the difference?, and others should allow for a great deal of mathematics.

The program outlined is not just a remedial lab view. It can be used within classrooms that are open, individualized, or traditional. It is not a specific method with set rules to follow. It is, hopefully, a sensible view of mathematics and a way to meet a child's needs in this subject.

"Par" - Puzzles + Arithmetic = Remediation

In all mathematics teaching situations, but especially in remedial classes, the ideal method of teaching is to proceed gradually from the concrete (operations with sets of objects) to the abstract (manipulation of symbols). First, utilize objects; i.e., manipulative materials. When the child shows that he understands the relationship between a specific numeral and its related number (of objects), the mechanics of writing the numerals can be introduced and the objects can be replaced eventually by pictorial representations.

After some simple number facts have been learned, reinforcement (commonly called "drill") can be provided in a variety of ways — e.g., number games, puzzles, etc.

The following exercises have proven very helpful in developing the abilities of underachievers, and fostering in them an appreciation of, and a desire to better understand the world of mathematics.

1. Introductory classroom activity to emphasize:

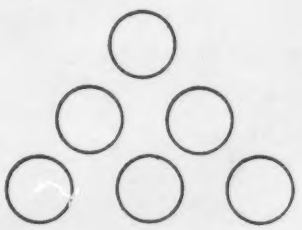
   (a) problem-solving strategies

   (b) number patterns and relationships

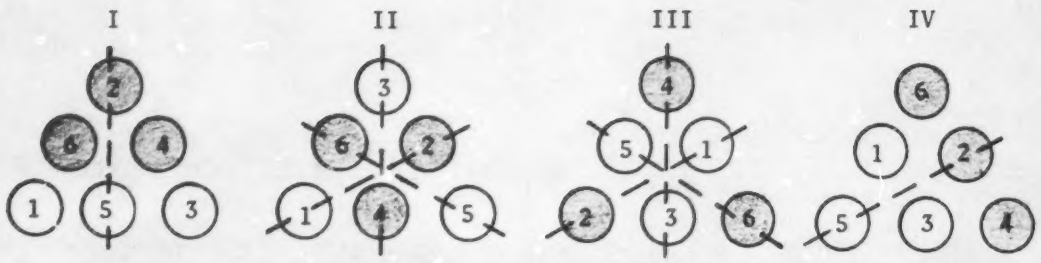
   (c) making predictions

   (d) symmetry

Using digits 1 through 6 (these digits could be written on pieces of paper to be manipulated), arrange them in an array as shown so that the sum of each side is 9. (When solved, arrange digits to make the sum 10, then 11, then 12.)



Solutions



side sum 9 side sum 10 side sum 11 side sum 12

Things that could be noticed:

Triangular array (I) has lowest numbers in corners while triangular array (IV) has highest numbers in the corners.

Triangular array (II) has odd numbers in corners while triangular array (III) has even numbers in the corners.

Triangular array (I) turned "inside out" is equivalent to (IV).

Triangular array (II) turned "inside out" is equivalent to (III).

The sum of the corners forms a sequence of multiples of 3 — i.e., 6, 9, 12, 15.

If even numbers are shaded, (I) and (IV) have one axis of symmetry while (II) and (III) have three axes.

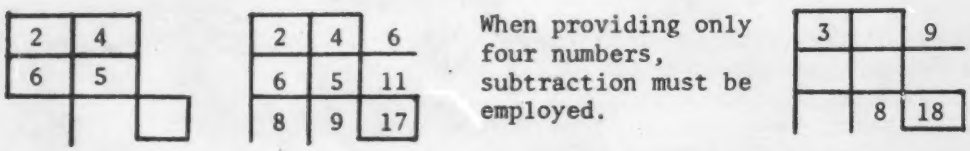
2. Using digits 2 through 7 in same array as above, find side sums 12, 13, 14, and 15. Compare these solutions with those found above. Predict some results if digits 3 through 8 are used.

3. Cross-number puzzles

(a) Addition

Cross-number puzzles, as the one shown below (illustrating, +, 2, 4, 6, 5), include six different addition examples

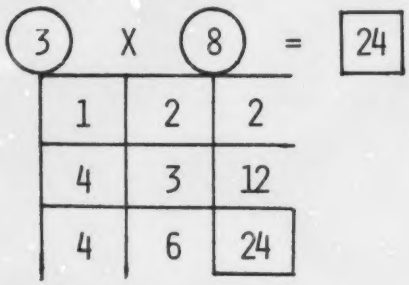
Given Answer



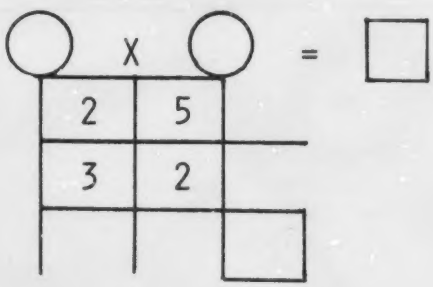
When providing only four numbers, subtraction must be employed.

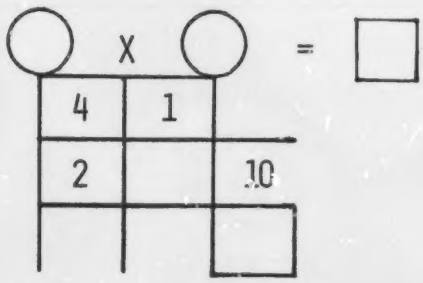
(b) The cross-number multiplication puzzle has "ears" at the upper corners. These "ears" are used to record the product of the two numbers on the diagonal.

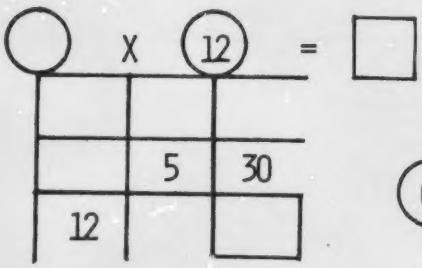
Example:

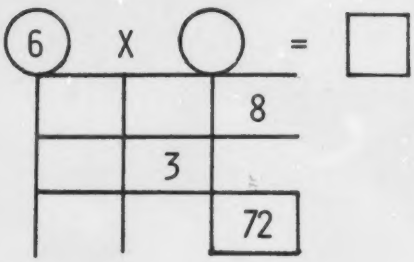


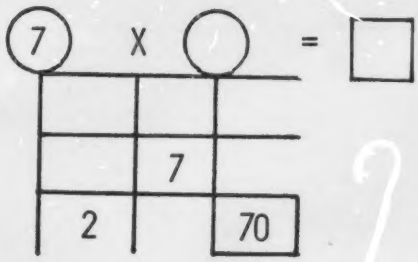
Solve the following puzzles:











(c) Variation of cross-number puzzle. The factors go in the circles and the product is written in the rectangle. The squares are used for recording. Here is a sample showing how the diagram is used in working out the product of 9 x 8. First, the 9, 8, and 72 are written in the appropriate places. Addends whose total is 8 are written vertically above the 8. Addends that total 9 are written horizontally to the right of the 9. See is you can tell how the other numbers are obtained. Work out another puzzle for 9 x 8 using different addends.

(9)	4	5	
45	20	25	5
27	12	15	3
72	32	40	(8)

Try working out a few other examples such as the following:

(a) 7 x 6
(b) 9 x 26
(c) 41 x 26
(d) 55 x 13

4. A palindrome is a word or phrase which is spelled the same when written forwards or backwards.

Examples: Otto, Mom, Hannah

"Madam I'm Adam"

"Live was I ere I saw evil."

Palindromic numbers: 121, 353, 18981, etc.

Every number has a palindrome associated with it. Start with any number. Reverse the order of the digits and add this reversal to the original number. If the sum is a palindrome, stop. Otherwise, reverse digits of sum and add again. Continue until you get a palindrome.

```
  238
  832
 1070
 0701
 1771
```

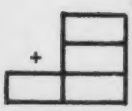
(a) Which numbers less than 100 are palindromes?

(b) Which numbers less than 100 take only one addition to obtain a palindrome?

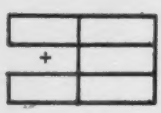
(c) How many additions are necessary to obtain a palindrome from 89? From 98?

5. SAD Game (Sum of All Digits)

Examples:
```
  3
+ 2
  5
```
3 + 2 + 5 = 10 SAD score



What is the largest SAD score you can get for example on the left? (one digit in each box)



What is the largest for this one?

Obviously, the SAD game could be extended, using several arrangements of squares.

6. Another problem in arranging digits:

Use digits 0 through 9, each once and only once, to make the addition example true.

Note: Twenty-one solutions to this problem were found, but there are probably many more. Here is one solution:

$$\begin{array}{r} 789 \\ +\ 246 \\ \hline 1035 \end{array}$$

7. Calendar Arithmetic

Use a calendar to answer all the questions.

(a) Select a 2 x 2 square of dates from the calendar. Find the sum of each diagonal. Record results. Does this always happen?

(b) Select a 3 x 3 square of dates from the calendar. Does the diagonal relationship hold? Find the sum of the center column; center row. Multiply center number by 3. Record results.

(c) Select a 4 x 4 square of dates from the calendar. Find the sum of each of the first three columns. Predict the sum of the last column. Find the sum of each of the first three rows. Predict the sum of the last row.

(d) Find the sum of numbers in any complete row (Sunday to Saturday). Divide by Wednesday's date in that week. Repeat for other rows. What happens?

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

8. Egyptian system of multiplying - based on doubling

Problem:

Rameses, a camel owner, decided to sell 12 of his camels to Aten. Aten agreed to pay six silver pieces for each camel. How many pieces of silver should Rameses get?

6 x 1 = 6
6 x 2 = 12
6 x 4 = 24
6 x 8 = 48
6 x 16 = 96

Solution:

A table was made as at the right. Start with 6 x 1 and keep doubling the factor until you reach a factor greater than 12.

8 + 4 = 12, so 24 + 48 = 6 x 12

Try these: (a) 15 x 16 (b) 24 x 9 (c) 18 x 31 (d) 84 x 23

9. Russian Peasant multiplication - doubling and halving

This is how a Russian peasant might have solved Rameses' problem. The peasant would have made a table starting with 6 x 12. The next entry in the table would be made by halving the 6 and doubling the 12. This process is continued until 1 is obtained on the left side of the table (remainders are dropped). Then the peasant would cross out any entry that had an even number in the left hand column, and add all the entries in the right hand column that were not crossed out. This sum is equal to 6 x 12.

6 x 12	~~6 x 12~~
3 x 24	3 x 24
1 x 48	1 x 48
	72

More examples:

~~28 x 56~~	27 x 13
~~14 x 112~~	13 x 26
7 x 224	~~6 x 52~~
3 x 448	3 x 104
1 x 896	1 x 208
1568 = 28 x 56	351 = 27 x 13

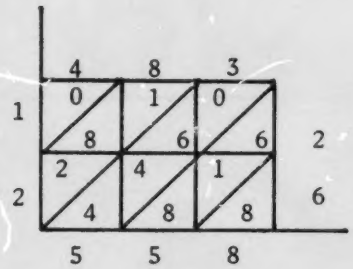
Try some of your own.

10. Lattice Multiplication

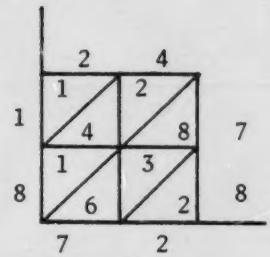
This method was used in most of Europe during the time of Christopher Columbus.

Directions: Make a grid as shown below. (Dimensions of grid depend on the number of digits in factors - e.g., 483 x 26 requires a 3 x 2 grid; 24 x 78 requires a 2 by 2 grid.) Partial products are entered in squares (digit in tens place separated from digit in ones place). Add partial products by adding numbers on diagonals, beginning with the digit in the lower right-hand corner, "carrying" as needed.

Examples:



483 x 26 = 12,558



24 x 78 = 1,872

11. Napier's Bones

In the 16th and 17th centuries in Europe, the great masses of the peasant working class had little, if any, education. They did not know the basic multiplication tables. John Napier, a Scottish mathematician, developed a mathematical aid that anyone could use to find the products of basic multiplication facts. He made pocket-sized multiplication rods, which he carried about and demonstrated. These rods became so closely associated with him that they were jokingly referred to as "Napier's Bones."

A set of Napier's bones is illustrated below. The first "bone" is the *Index* bone. The first digit at the top of each bone is another index factor.

For single digit multiplication (such as 6 x 7) the bones are used the same way as a multiplication table. To find the product of 6 and 7 you need only two bones — the index bone and either the 6 bone *or* the 7 bone.

X	1	2	3	4	5	6	7	8	9
1	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9
2	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8
3	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7
4	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6
5	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5
6	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4
7	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3
8	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2
9	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1

7 x 6

X	6
1	0/6
2	1/2
3	1/8
4	2/4
5	3/0
6	3/6
7	4/2
8	4/8
9	5/4

6 x 7

X	7
1	0/7
2	1/4
3	2/1
4	2/8
5	3/5
6	4/2
7	4/9
8	5/6
9	6/3

X	4	3
1		
2		
3		
4		
5	2/0	1/5
6		
7	2/8	2/1
8		
9		

To multiply 57 x 43, the index bone is used along with the 4 bone and the 3 bone.

The bones yield only the partial products, so it is necessary to do some addition to arrive at the total product.

57 = 50 + 7, so 57 x 43 = (50 + 7) x 43 = (50 x 43) + (7 x 43)

50 x 43 = 2150

7 x 43 = 301

2451 relate to lattice mult.

	4	3	
2	2/0	1/5	5
4	2/8	2/1	7
	5	1	

NOTE: The first factor is always represented by the index bone (regardless of how many digits). Make your own set of "bones" and try some examples of your own.

Regrouping in Subtraction

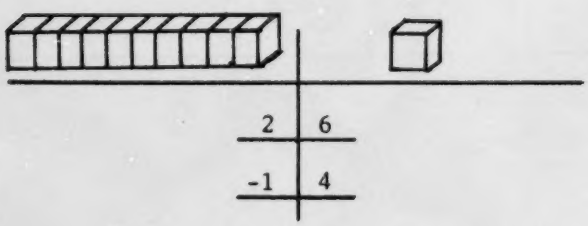
In order to best utilize the following activities, the students should be able to:

- identify the ones, tens, and hundreds columns in a given numeral
- complete a two-digit or more subtraction problem that does not require regrouping
- represent a given number in various ways in terms of place value using base 10 blocks or other manipulative materials; for example, regroup 4 tens and 3 ones as 3 tens and 13 ones

The materials that are the most helpful in teaching regrouping in subtraction are the base ten blocks. If these are not available, popsicle sticks, tongue depressers, or anything similar can easily be used by making bundles of 10 for the tens place and loose sticks for the ones.

The following is a suggested procedure to follow:

1. Using the blocks, have the students complete a problem that does not involve regrouping and record their answer. Depending on the student's level, mark the columns with pictures of the blocks, tens and ones, or no marking at all. Whichever is used, the move is towards unlabeled columns. (See Figure 1)



2	6
-1	4

TENS	ONES
2	6
- 1	4

26
-14

Figure 1

2. When the students can do the above independently, give them a problem involving regrouping. In the example shown (Figure 2), having represented 34 as 3 tens and 4 ones, the student will quickly note that one cannot subtract (or take away) 6 ones. Having the 4 ones right in front of him also removes the student's temptation to reverse and subtract 4 from 6 (a common error). In this example, we have established with the student that 34 is more than 16 and so we should be able to subtract. Then ask him for suggestions as to what we can do in order to complete the subtraction. Frequent responses are exchanging a ten for ones or cutting a ten rod apart. Some students will suggest trading in all the tens for ones, but they soon realize that they need only trade in one ten. If the student doesn't suggest the trade on his own, ask him about this possibility. Emphasize the two ideas: (1) that 1 ten = 10 ones (some students will just match the blocks and trade, not

noticing this equality) and (2) that the ten ones are added to the ones we already had, resulting in 14 ones. Then have the student complete the problem by first taking away 6 ones and then 1 ten. The answer should then be recorded.

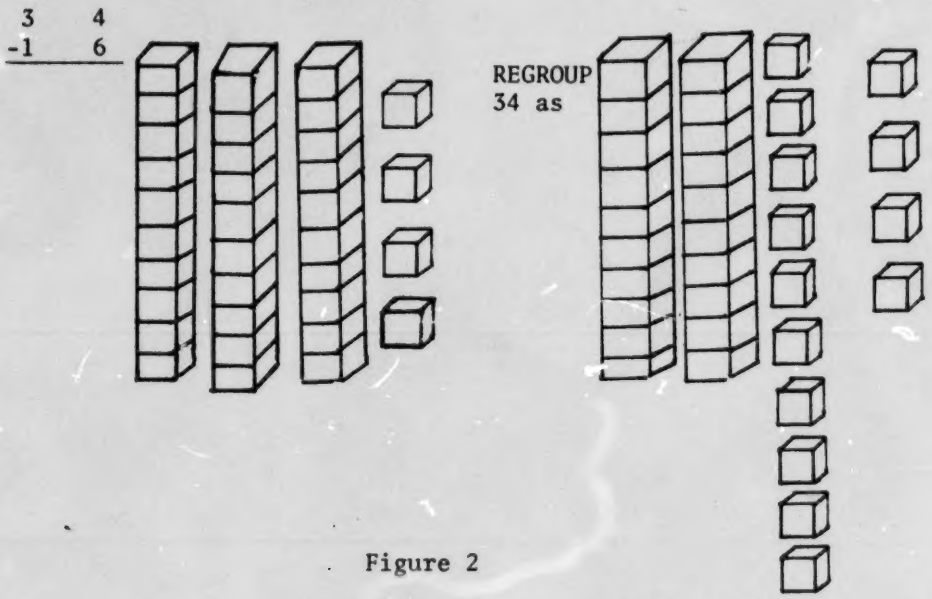


Figure 2

3. When the students can independently complete examples such as the above, introduce them to actually recording the regrouping process. Experience has shown that many students have run into problems by recording the regrouping in this manner:

$$\begin{array}{rr} 2\not{3} & 1_4 \\ -1 & 6 \\ \hline \end{array}$$

They don't know that they can put the one in front of the four to show 10 + 4 = 14, so it becomes a rote process. By recording the regrouping as follows, the students are mentally adding 4 to 10 so

$$\begin{array}{rr} 2\not{3} & 1\not{4} \\ -1 & 6 \\ \hline \end{array}$$

that they understand why they get 14. Some students will notice that they can use the other notation, and will use it as a normal short cut.

4. In providing practice problems, always include problems that don't involve regrouping to prevent the generalization that hereafter one *has* to regroup in every subtraction problem. Using the blocks enables the students to make easily the distinction as to when regrouping is necessary. Use the blocks along with the recording until the students no longer feel the need for them.

5. After the students have become familiar with the base ten blocks (or bundles of sticks or whatever was used), materials of a more abstract nature could be introduced. There is a commercially made chip trading game that is good for this, but don't let the lack of funds stop your use of the idea. All that is needed are a few different colored poker chips or even cut out poster-paper pieces. Let one color represent ones, another tens, another hundreds, and another thousands. Before doing actual subtraction (or addition) examples, it would be best to familiarize the students with the color values of the chips first. This can be done easily by playing a simple dice game. A player throws the dice (or die) and a banker gives him that same value in chips. Suppose a yellow chip represents a one, a red chip counts as a ten, and a blue chip is worth a hundred. Each player takes his turn and collects chips from the banker. The only rule is that a player can never have more than 9 chips of the same color. As soon as a player gets 10 of one color, he or she must trade them in for the equivalent chip.

   Subtraction (as well as addition) examples can be worked out using trading chips in a way similar to that described with base ten blocks above.

   Note: Chip-trading activities very closely resemble the use of money; e.g., think of the yellow chip as a penny, the red chip as a dime, and the blue chip as a dollar. This naturally leads us to money activities which are presented in the next article.

Money Games

Money games are a good way to introduce mathematics to the young child. From a very young age, in present-day society, children are exposed to money-management concepts. As adults, we do not always realize that this number exposure is taking place. What better way, then, to introduce number concepts - base ten system, decimal notation, etc.- than by using something already familiar to the student? If play money is used, which of course is more prudent, be sure to use the types that resemble real money as much as possible. There are several producers that sell money kits.

Activity 1 Coin Trading Games Number of players: Up to 5 plus banker

Game 1: Trading pennies, dimes, dollars

Objectives:
- Reinforcement of coin values and equivalencies
- Coin manipulation
- Set recognition and computation (on dice)
- Reinforcement of place value concepts

Materials:
- Teacher made gameboards - each board has three columns labeled: pennies, dimes, dollars
- Two dice
- Toy pennies, dimes, dollars

Procedure: For the first game or two the teacher may have to be the banker. After this, one child becomes the banker and the others are players. Each child, in turn, rolls two dice and computes the appropriate sum. The banker gives out pennies in the amount called out by the player. The player puts pennies in the penny row on the gameboard. He or she is not allowed to have more than 9 pennies on the board. When the player has 10 pennies, he or she must trade the banker for a dime and put it in the dime row. The first player to reach 10 dimes trades for a dollar and is the winner and becomes the next banker. During the game the teacher should occasionally say, "stop trading," and ask each player to name the value of the coins on his or her board and the total amount of money he or she has.

Game 2: Trading pennies, nickels, quarters

This game is played the same as the above game, but not more than four coins are allowed in each row. Game boards are similar to those in game 1, except the columns are labeled: pennies, nickels, and quarters. The first student to have 5 quarters is the winner, and must trade 4 of his quarters for $1.00. Thus, the winner has $1.25. Both of the games could have several variations.

Activity 2 Box of Shapes Activity Number of players: Entire class

Objectives:
- Addition skills reinforcement
- Use of addition as an operation for totaling
- Geometric constructions and relationships

Materials:
- A box filled with construction paper triangles, squares, and parallelograms scaled to fit together
- Background paper
- Paste or glue

Procedure: Each shape should have a price written on it, such as 1¢, 19¢, etc., depending upon the performance level of the children. Students should paste shapes on background paper, making different patterns. They may need guidance with this. When they have completed their pattern, or patterns, they are to total the prices in each pattern and write it next to the pattern. At the bottom of the construction sheet, they may calculate the cost of all the patterns and add the amounts together to find the "total cost."

Activity 3 Store and Shopping Activities Number of Students: Small group

Objectives: The objectives of these shopping activities cover a wide range, including:

- Practicing in coin usage
- Decimal notation in money problems
- Place value reinforcement
- Computation practice
- Improvement of problem-solving techniques
- Practical experience with measurements
- Aid in ability to organize material

Materials:

- Play store (if none is available, make a simple one of stacked, large, heavy-duty cartons)
- Empty, labeled, clean, unbreakable food containers with prices labeled on them
- Toy money
- Lined paper
- Adding machine, cash register or calculator
- Ads from newspapers
- Catalogs of all sorts
- Activity cards indicating tasks to be performed

Procedure: In the following activities, it is important that the teacher observe each child's work in order to be sure that correct operations are being used and that numerals are being written and placed correctly.

The activities for shopping could be sequenced as follows:

(1) Simple buying activities using toy coins, progressing to activities in which the children make lists, total prices on lined paper, and check totals with a storekeeper. When children are ready for this totaling, which will involve correct placement of numerals, they will have to be supervised carefully.

(2) Give shopping lists to the children and have them find the total cost of their lists. In this way, you can make each list compatible with the child's ability. During the course of this activity, make use of comparisons: "Who spent more...or...?" "How much more?"

(3) Give each child a fixed amount of money, and see how many things he or she can purchase with it in accordance with displayed newspaper ads or catalogs.

The above activities are merely brief suggestions. The teacher could easily make many exciting and worthwhile extensions of these to motivate the students to fulfill all the objectives listed.

A Visual Sequence for Teaching Fractions

Some pupils experience difficulty when asked to write a fraction to describe a part of a whole. One reason for the difficulty is that the given model usually does not match its fraction numeral. Model A shows a whole figure in which one part has been shaded and three parts have not. While this suggests that the numerals *one* and *three* be included in the fraction numeral (one for the shaded and three for the not shaded), this is not the case, as the fraction name for the part is 1/4.

Model A

In the notation for model A, the bottom numeral names the fraction family (fourths); the top numeral counts the number of parts of this family that have been identified through shading (one). Pupils must, in effect, count the shaded part twice in order to arrive at the fraction name. A sequence of prerequisite activities is helpful to pupils in understanding the fraction notation 1/4 as it relates to the model. The activities are based on a model which does match its fraction numeral initially and then goes to shading.*

The following sequence can lead to an understanding of the notation for fractions:

(1) Development of the concept represented by the bottom numeral: *fraction families*

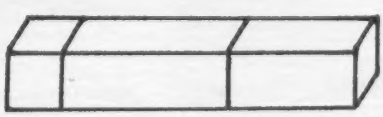
(2) Development of the concept represented by the top numeral: *parts*

(3) *Transition* from matching to shading

Model B

Model C

Model D



Model E

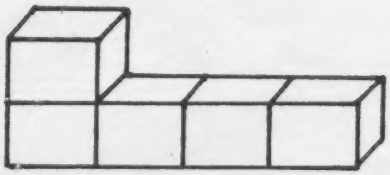
* The model is presented in *Title IV-C Fractions: An Early Approach*, a primary program developed by the Rochester City School System's Mathematics Department and is used as a basis for teaching adding, subtracting, and renaming of fractions.

Step 1: (See models B, C, D, E.) Pupils build these models (or they may be teacher-made) and examine each to determine whether or not the fourths family is represented. Reasons are then stated by the pupils. It is determined that B is the correct model.

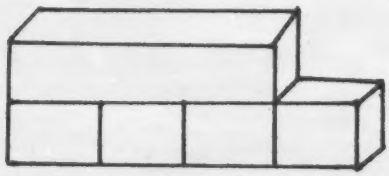
$\frac{}{4}$

Model B Fourths Family

The notation /4 is used to represent fourths as shown in the model above.



Model F



Model G

Step 2: (See models F and G.) Pupils build these models (easily done with rods) and write the notation. The top numeral counts the number of parts of a fraction family (fourths family used in both cases) that have been matched. In model F, the number of fourths that have been matched is one, so the fraction notation is 1/4. In model G, the number of fourths that have been matched is three, so the fraction notation is 3/4.

Part Sketch of fourths family

Model H

Step 3: (See model H.) Pupils are given a worksheet showing a sketch of the fourths family. They are directed to match a manipulative part with the sketch, and then to shade the area where it matches. Since the shaded part matches the manipulative part, it can then be described by the notation 1/4.

A Space to Carry in Simple Addition and Multiplication Examples

Addition and multiplication present similar problems when numbers must be "carried" from one place to the next.

Addition

First: Use a double line at the bottom of each problem so that horizontal and vertical problems use the same

symbols and are read the same way. The double line in the vertical problem is read as "equal" and resembles the equal sign in a horizontal problem.

$$\begin{array}{r} 8 \\ +7 \\ \hline \\ \hline \end{array}$$

Same as

$8 + 7 =$

Second: Rather than ask a student to write a number sum in the reverse order (the way it is usually done), write the "carry number" first - i.e., when recording the sum of 9 plus 6, the 1 in the ten column is written first, then the 5 in the ones place. This tends to minimize the error of a student writing the 1 in the ones place and carrying the 5.

Third: When the carry number is noted at the top of the problem, the distance between the parts of the number permits incorrect placement.

If the equal lines are separated slightly, a work space is created to place the carry number so that the parts of the number are close together.

$$\begin{array}{r} 15 \\ 17 \\ +19 \\ \hline +\ 2 \\ \hline 51 \end{array}$$

Multiplication

Multiplication is unique in that it is a combined multiplication and addition algorithm. With the carry number above the problem, the following mistakes are common:

- The numbers are added first, then multiplied,
- The carry number is incorrectly placed,
- The carry number is used as a factor instead of an addend.

If the work space between the equal line is used for the carry numbers, only multiplication is done with all the numbers above the equals lines, and the "in between" lines (double equal sign) can be utilized for carry numbers which are to be added only. Then, each operation has its own location.

$$\begin{array}{r} 13 \\ \times\ 7 \\ \hline +2 \\ \hline 91 \end{array}$$

To ensure that the "in-between numbers" or "carry numbers" are added, a "+" sign could even be inserted.

Note: This system of using a double line (equivalent to an "=" sign) at the bottom of a vertically written example is to be used only as a "crutch" until the traditional algorithm can be utilized.

Grid Paper Computation

Sometimes remediation in computational skills simply involves the problem of neatness - keeping numerals in neat and accurate columns.

I can add and subtract oh so fine
All those facts are not troubles of mine
But my columns get bends
I mix ones with the tens
Oh you numbers, please stay in line!

What can be more frustrating for a child than to carefully add or subtract all the facts in a multi-digit problems and arrive at an answer only to find that because his columns of numbers were not clearly defined, he misplaced a number, added it twice, or omitted it entirely? The arrival at a correct answer often depends as much on the neatness of the written problem as it does on the knowledge of the facts or operation. The teacher and student have a common problem: How can I avoid the messy situation of crooked columns so that I can get on with the more important objectives of math facts, regrouping, and accurate answers?

A simple solution to the neatness problem exists in changing the kind of paper used. Lined paper, as used today for mathematics computation, offers no guide for children writing numerals in columns. The guiding lines on paper are horizontal because we read and write from left to right. But we usually compute mathematics problems vertically. How then, can we utilize existing technology to solve this problem? Simply by using graph paper (or grid paper). Now, there are rows (as on ordinary paper) to guide the horizontal numerals, and also columns to be used as a guide for vertical computations.

There are two simple rules for every mathematics assignments:

(1) Digits may be written within the squares only.

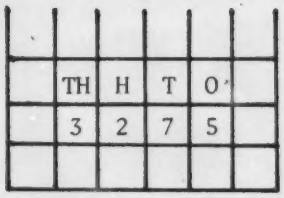
(2) One and only one digit from any given numeral may be written in a single square.

The benefits from this system are significant:

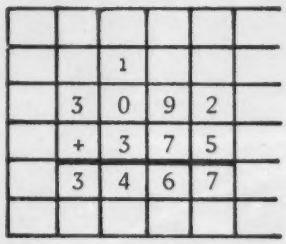
- There is no question as to which digits of a numeral are to be added together because they are directly below each other.

- There is only one place to put the answer to each column of numerals and that is in the square directly beneath it.

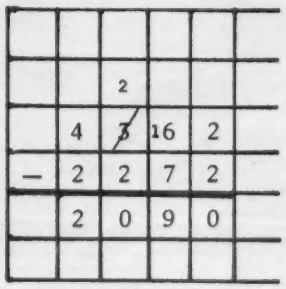
- Discussion of the operations in the squares leads to an easy inclusion of a discussion of the place value of each number in each square. Children could also label each column before computing (see example at the right). For example, 3275 means 3 thousands + 2 hundreds + 7 tens + 5 ones.



- The squares serve as a constant reminder for addition regrouping, when necessary. When the 9 tens and the 7 tens are added, there is no place to write the 16 tens. Since only one of the digits can be placed in the tens square, a 1 is placed in the hundreds square to represent the 10 tens (100).



- The squares provide a clear picture of the placement of the digits when regrouping in subtraction. The child tends to focus on the squares and therefore is more careful where he takes and places his numbers. Since 7 tens cannot be subtracted from 6 tens, we must bring one of the hundreds into the tens square as 10 tens. This action creates a temporary excess of numbers in the tens square but makes subtraction possible.

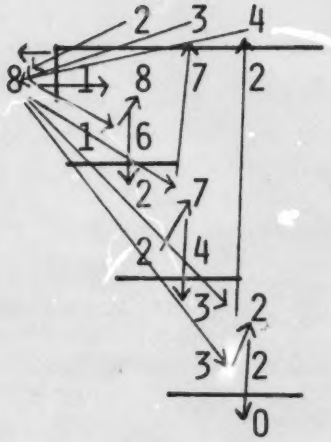


This method of using graph paper can obviously be utilized in performing multiplication and division examples as well.

If grid paper is not available, consider using regular lined paper, turned 90° to permit column guides.

The Need for Math Reading Skills

Does the sketch on the following page look like the work of a neat but frustrated mathematics student? On the contrary, these lines represent the different directions the eye must follow in order to complete this fairly simple division problem. This is an example of the "reading" which occurs in mathematics. Many students are unprepared for this type of reading and that is where their deficiencies lie. They become labelled "slow learners" in need of remediation.



"Learning to read in the language of mathematics is a crucial competency needed by a large majority of the students in our schools. Yet, the usual reading and mathematics programs do not, in general, provide the sorts of activities necessary to acquire this competency."[1]

The preceding quote was taken from an article by Hater, Kane and Byrne on building reading skills in the mathematics class. In it, they identified thirteen skills that are used in reading the language of mathematics. Some of those skills will be presented briefly in the paragraphs which follow.

Knowing what to read next is a skill the authors identified. In mathematics, unlike written English, symbols may be read in a number of directions. Left to right, as you are now reading, is not always the case in mathematics. Symbols may be read right to left, diagonally, top to bottom, bottom to top and so on. A simple set of symbols may be read in many different ways. For example, $2 + 5^2$ may be read "two plus the square of five" or "five squared plus two." Several other readings are possible for the same set of symbols.

Not only can a set of symbols be verbalized differently but an oral expression may be written in different ways by different students. An example such as "six times eleven" may be written as:

6×11 $\qquad$ $\begin{array}{r} 11 \\ \times\ 6 \\ \hline \end{array}$ $\qquad$ $6 \cdot 11$ $\qquad$ $6\,(11)$

Division lends itself to an even better example with the possibility of different students using either a fraction bar, the division sign (÷) or the division symbol ($\overline{)}$) to represent a problem. As these examples point out, oftentimes in mathematics, there are several symbolic representations for the same expression. Students must learn to feel comfortable with the

[1] Mary Ann Byrne, Mary Ann Hater, and Robert B. Kane, "Building Reading Skills in the Mathematics Class," *Arithmetic Teacher*, Vol, 21, No. 8 (December, 1974), p. 668.

symbols of mathematics and not be confused by their variety. M. A. Byrne identified 153 different symbols, exclusive of the alphabet, which appear in mathematics books for grades four through twelve.[2] Although many students will never be familiar with all these symbols, it is important that they understand and are able to use the ones introduced to them.

Key words also play a role in the reading of mathematics. Although the two statements below differ by only one word, the difference in their mathematical meaning is great. (See Figure 1.) The key word of "times" must first be recognized by the student, then understood and finally symbolized to solve the problem.

SEVEN MORE THAN EIGHT

SEVEN TIMES MORE THAN EIGHT

Figure 1

Words with many meanings are part of another skill the authors identified. Some words used in mathematics such as "quotient," "percent," and "decimal" have specific mathematical meanings. As words, they do not lead to confusion for most students. Other words which sound and look like words in everyday speech are more of a problem. Words like "plane," "root," "odd," "union," "power," and so on, can be cause for confusion to some students. Since they are more familiar with their everyday meanings than their mathematical ones, the latter must be stressed for reinforcement.

This article, by no means, encompasses all the skills needed to read mathematics. There are many more too numerous to mention here.[3] However, it must be understood that none of these skills is isolated and they are all dependent on each other. The mathematics class is the place to learn how to read mathematics by combining the skills used to read while actually reading and working on mathematics.

A Structural Approach to Multiplication

One of the main values of having children study multiplication is to enable them to use it efficiently in problem solving in their everyday lives.

In order to gain efficiency in problem solving and computation, a generalized procedure for carrying out an operation must be learned. The child must become proficient with traditional algorithms in order to understand and appreciate better the impact and utility of the widely used electronic calculator and its vast implications in modern problem solving. Obviously, efficiency is a consideration in computation. Therefore, we eventually teach children to memorize algorithms. These algorithms must extend beyond

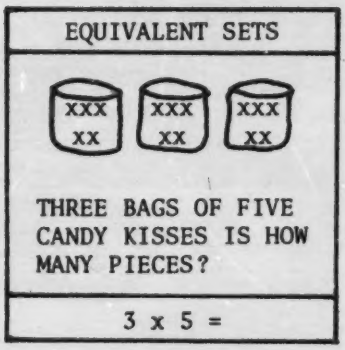
[2] Ibid., p. 665.

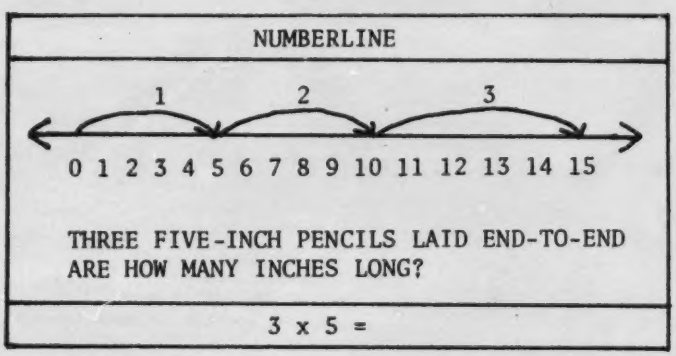
[3] Another resource is the New York State Education Department publication, *Improving Reading-Study Skills in Mathematics.*

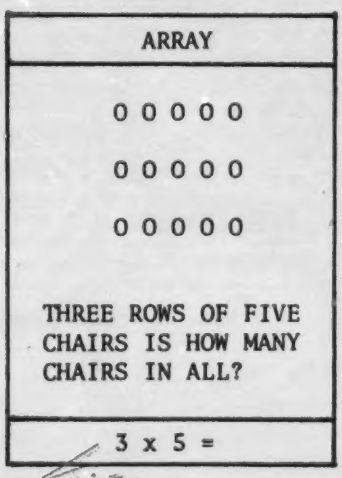
the written examples - i.e., they should also include electronic calculators and mental arithmetic algorithms for all students. Children need practical experiences to determine which algorithm is best used. Most everyday applications are best solved by mental arithmetic, since we do not have other materials, such as pencils and calculators with us all the time.

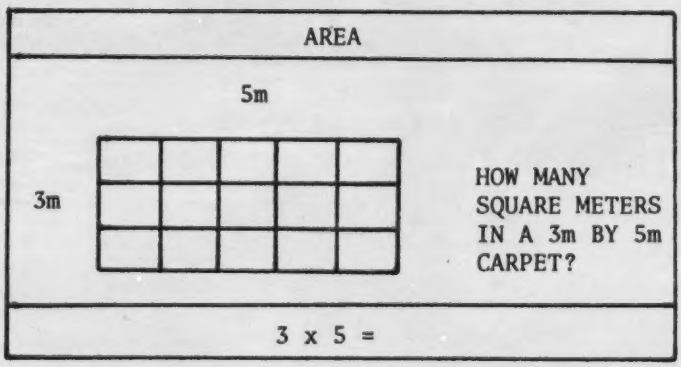
In meaningful teaching of arithmetic, computation develops out of problems, and is used to help solve other problems efficiently. Problems are not just work problems in the textbook placed at the end of the unit on multiplication. They are used throughout the unit. The solving of problems increases the understanding, usefulness, and purpose of computation, and, as computational proficiency increases, the student's ability to solve problems increases proportionally.

One often overlooked aspect of teaching multiplication is the teaching with concrete or pictorial models. Models are used to help translate words or real life situations into mathematical symbols (sentences or algorithms) and conversely, to translate mathematical symbols into classes of applications. Briefly, the models of multiplication are equivalent sets, number line, arrays, cross product, and area. Each of these five models is illustrated below.









CROSS PRODUCT

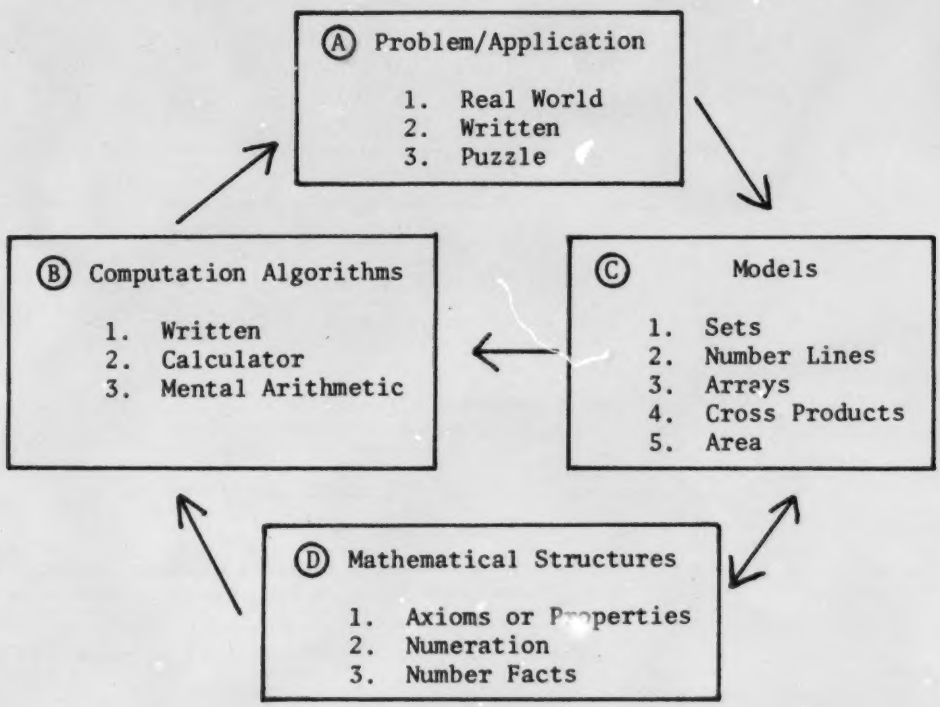
SKIRTS	BLOUSES: White	Red	Blue	Green	Yellow
Brown	0	0	0	0	0
White	0	0	0	0	0
Blue	0	0	0	0	0

HOW MANY OUTFITS CAN YOU MAKE WITH THREE SKIRTS AND FIVE BLOUSES?

3 x 5 =

Modern mathematics places a great deal of emphasis on mathematical structure. This structure is also a very important aspect of learning multiplication. The structure for multiplication can be broken into three areas: (a) axioms or properties, (b) numeration system, and (c) the number facts. The three most important properties for multiplication are the distributive, commutative, and associative properties. Place value is the area of greatest difficulty in the structure of the numeration system. Number facts remain an important part of the structure that need first to be understood and then memorized.

These four aspects of the meaningful teaching of multiplication are diagrammed below:



As the diagram illustrates, computation is central to this whole process. Written algorithms hold a special place in developing the child's facility with multiplication because the child can "see" the structure applied in the algorithms, relate the models to algorithms, and check his work against the problem or application. (Calculators already incorporate the structure and unless carefully used, "hide" the structure from the child, and become magic black boxes.)

A few sample transitional algorithms should illustrate how they tie together the whole concept of multiplication, even if the child's understanding is presently on a low level.

(1) Three teams of 11 girls are going to a track meet. How many girls in all are going to the meet?

	10	+	1
	x x x x x x x x x x		x
3	x x x x x x x x x x		x
	x x x x x x x x x x		x

$$11 \times 3 = 10 + 1$$
$$\underline{\times\ 3}$$
$$30 + 3 = 33$$

(2) One hundred fifty-six passengers on a plane paid $350 each for a trip to England. How much did they pay altogether?

(a)

ten thousands	thousands	hundreds	tens	ones	
		3	5	0	$54,600 was paid altogether. (used column addition to hold place value; multiplied one digit at a time)
	x	1	5	6	
				0	6 x 0 =
		3	0	0	6 x 50 =
	1	8	0	0	6 x 300 =
				0	50 x 0 =
	2	5	0	0	50 x 50 =
1	5	0	0	0	50 x 300 =
3	5	0	0	0	100 x 350 =
5	4	6	0	0	

(b)
$$\begin{array}{r} 350 \\ \times\ 156 \\ \hline 2\,1^{3}00 \\ 1\,7^{2}5\,00 \\ 35000 \\ \hline 54{,}600 \end{array}$$

$54,600 was paid altogether.

(Partial products have zeros added and the crutch of carry numbers is used.)

(c)
$$\begin{array}{r} 350 \\ \times\ 156 \\ \hline 2100 \\ 1750 \\ 350 \\ \hline 54{,}600 \end{array}$$

$54,600 was paid altogether.

(traditional algorithm)

These algorithms are discussed and the children use them as they please. Discussion will lead to the advantages of the standard algorithm, although "crutches" will usually persist for a longer time. A child should be encouraged to drop the crutches when they are no longer needed.

When checking computation papers, look for error patterns and then work with the individual child to correct these error patterns. Examples of error patterns are given below:

Margaret

(a)
$$\begin{array}{r} {\scriptstyle 1} \\ 306 \\ \times\ 25 \\ \hline 180 \\ 72 \\ \hline 900 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 3} \\ 208 \\ \times\ 45 \\ \hline 140 \\ 112 \\ \hline 1260 \end{array}$$

(c)
$$\begin{array}{r} {\scriptstyle 2} \\ 790 \\ \times\ 35 \\ \hline 395 \\ 237 \\ \hline 2765 \end{array}$$

Jim

(a)
$$\begin{array}{r} {\scriptstyle 3} \\ 36 \\ \times\ 6 \\ \hline 366 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 2} \\ 53 \\ \times\ 7 \\ \hline 491 \end{array}$$

(c)
$$\begin{array}{r} {\scriptstyle 2} \\ 49 \\ \times\ 3 \\ \hline 187 \end{array}$$

Margaret sees zeros as "place holders." Since they are not numbers, she does not bother with them and regroups her tens to the hundreds place, where there is a "real" number.

Jim is mechanically applying the crutch by adding it to the tens column before he multiplies.

Both children are showing evidence of lack of basic computational understanding. Telling Margaret to multiply by zero and add the regrouped tens, or telling Jim to place his crutch below the line to remind him to add may

be short term solutions, but they will most likely cause greater problems later. It would be best to go back and improve their understanding of both the algorithm and the structure that underlies it. Their future growth and confidence depends upon this course of action.

Proper diagnosis and remediation in multiplication or any other area of mathematics is not cut and dried. It involves a sensitive teacher interacting with the child, and the child working with all aspects of multiplication. Computational efficiency followed by a few word problems is not a satisfactory approach. All aspects of multiplication must be interrelated with computation taking its proper place in the overall program.

The Electronic Calculator in Remedial Mathematics

Introduction

Calculators are here to stay. Children are fascinated by these incredibly small, but powerful devices. What should their role be in remedial mathematics? Despite the simplicity of these units, they are extremely powerful, if you know how to use them. There are plenty of procedures and tricks that will enable you to get more out of them than the engineers who designed them ever envisioned. However, there is an inherent danger in using electronic calculators in remedial mathematics programs. Calculators hide the structure of algorithms. This can be overcome by teaching the structure of the algorithm first. Then test the student in the usual way. After that, teach the student to perform the same operations on the calculator to check his/her answers already obtained by using paper and pencil methods.

In the hands of an imaginative teacher, the use of the calculator can alleviate many of the problems that "below grade level" students encounter.

The following problems were designed not only to manipulate numbers on a calculator, but to also complete a sentence or answer a question. Follow the simple directions and watch the surprised faces of the students as the puzzles unfold.

Directions

1. Solve each problem. Write the digits in the boxes, one digit in each box.

2. Enter the circled digits into the calculator in the order in which they appear. Decimal points in circles must also be entered.

3. TURN THE CALCULATOR UPSIDE DOWN AND READ THE ANSWER TO THE QUESTION.

(1) WHAT DOES MR. RIVERA FIND IN HIS MAIL EACH MONTH? _____.

(a) How much do three pens cost at $1.98 each?

(b) Donna has read one-third of a book which contains 231 pages. How many pages has she read?

(c) John has $50. Susan has $36.32. How much more money does John have than Susan?

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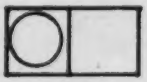
(2) IN ORDER TO START A GARDEN, YOU MUST HAVE _____.

(a) Juan received the following grades on his mathematics test: 80, 90, 75, 68, and 47. What was his AVERAGE grade?



(b) How many years are in 1-1/2 centuries?

(c) What is the perimeter of a rectangle whose length is 13 meters and whose width is 9.5 meters?

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(3) WHAT SHOULD YOU NEVER HEAR IN A COURTROOM? _____.

(a) Ruth has $15. Pencils cost $.06 each. How many pencils can Ruth buy?

(b) Cans of string beans are on sale 3/69¢. How much will 21 cans cost?

(c) Angela's brother is 8 years old. She is 9 years older than her brother. How old is Angela?

---

(4) IN THE SUMMER, YOU SHOULD BE VERY CAREFUL OF _____.

(a) How far will a train traveling at 57 miles per hour go in 9 hours?

(b) Wanda drove 98 miles on Monday, 106 miles on Tuesday, and 114 miles on Wednesday. How many miles did she drive in all?

Nature's Mathematics

The outdoor classroom can provide many of the experiences to make mathematical/geometrical symbols and processes meaningful and relevant. The lessons and activities suggested below allow for a wide range of student abilities and grade levels. With proper preparation and modification of activities, students become more sensitive and curious and will have a better understanding of the mathematical/geometrical aspects of their environment. Each success could be a rewarding experience for both student and teacher.

Matching Shapes

To have children begin to realize some of the basic geometric shapes in nature, the teacher might start by taking the children out on a leaf hunt. A discussion might follow regarding some simple leaf names, similarities among the leaves, the purpose of the leaves, etc. The teacher could then choose a few basic leaves and trace each leaf onto a card. Each leaf is mounted onto another card of the same size, and these pairs are kept in a box. The children must match the real leaf to the outline of the same kind of leaf, and be able to discuss the certain attributes of the leaves that helped them to match the pairs. The name of its tree could also be included for some children who read.

Prove It

This activity is one which may be used for reinforcement of the counting numbers, geometric shapes, or tree identification as well as stimulating an awareness of the immediate world around the children. The players sit in a circle, each with three counters. One will start by saying, "I see one robin" (or a geometric shape, or a type of tree). The next one says, "I see one robin and two ants." A third adds on three other objects, and so on.

At any time someone can say, "Prove It." If someone says something he cannot prove, he forfeits a counter to the challenger. If the student can prove it he or she receives a counter from each challenger. The game ends at any point in time, and the player with the most counters wins.

Scavenger Hunts

Children could use this activity as an aid in learning their counting numbers. Often, children cannot associate the number symbol with the meaning of that number. After a search in which the student might be seeking a specific number of objects, for example, four - a child should obtain a realization of the meaning of four.

If an article sought should not or cannot be removed, the student must be trusted to identify and declare that he actually did see it.

The following are suggestions for such scavenger hunts:

(a) People Pollution Pickups:

bottle cap	string
bottle	wire
empty can	foil
pop top tab	paper products
candy bar wrapper	cellophane
match book	plastics

(Note: warn students not to handle rusty or broken objects.)

(b) Articles plentiful in nature:

feather	round rock
acorn	bone
dandelion	berry
pine cone	shed bark
red leaf	four leaf clover
shiny rock	nuts

Tell Temperature From Cricket Chirps

Count the number of chirps in 14 seconds. Add 40 to this number and the sum is the temperature (Fahrenheit). This obviously gives children practice working with time, temperature, adding, data collecting, observation techniques, and natural phenomena.

Discovering Pi

This activity brings the children to a realization of a common mathematical constant - Pi. The students will need a measuring tape, paper, and pencil. Ask them to measure the circumference and diameter of as many "round" objects as they can in a designated time period, and keep a chart like the following:

OBJECT	CIRCUMFERENCE	DIAMETER	
car tire	78 inches	25 inches	

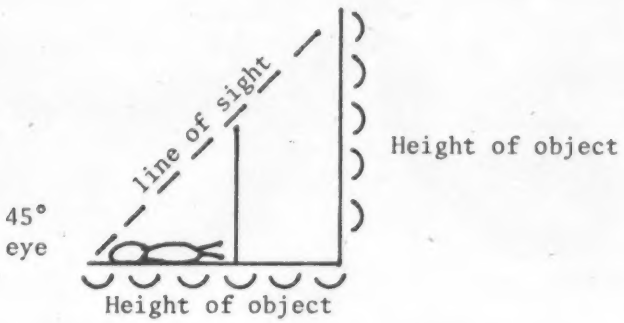
After the students have collected a certain amount of data, call them together and discuss the figures and ask if there is any mathematical relationship between the circumference and diameter of the objects. Have the students divide the circumference by the diameter and talk about the answer they got for each object. When they have talked about this constant, tell them that this number found in nature is called "Pi" (written π), and that we can use that number in calculations.

Finding Height

There is something in the mind that really motivates children to want to know how high a certain tree, building, or pole is. To capitalize on this

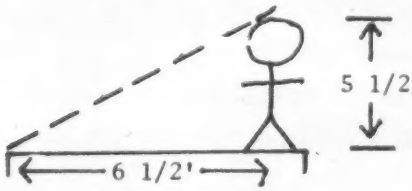
compelling force, two of the easier methods of finding heights will be explained. The first can be introduced by talking and reading about Paul Bunyan and the lumberjacks (which can be found in the "Tall Tales" section of most basal readers), and how they determined the height of trees.

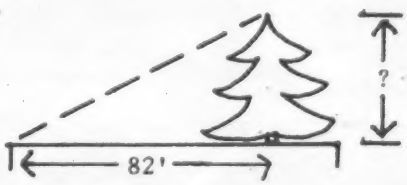
The teacher will need a stake that is quite long. Mark on it the height of a volunteer from the class with a bright piece of tape or marker. Have the student pick a tree and find a distance from the base of that tree which seems to be the same as the height. The student should lie on his or her back (a plastic sheet would be useful) and another student will need to hold the stake vertically at the feet of the person lying down. The student lying on his or her back will have to move either closer to or farther from the tree until the top of the tree lines up with the mark on the stake. When this is done (keeping the stake at the bottom of the student's feet at all times), the height of the tree is the same as the distance from the base of the tree to the student's head. See diagram below.



Did you know that the longest shadow in the world is cast by a mountain in the Canary Islands, and is over 150 miles long in the morning and evening? Just try and use pacing to measure that distance! To do this we could use the "shadow method" of finding heights. This would involve proportions, comparing the shadow length of a person (whose height you know) and the shadow length of the object.

Example: $\frac{\text{person's height}}{\text{person's shadow length}} = \frac{\text{tree's height}}{\text{tree's shadow length}}$





$\frac{5.5}{6.5} = \frac{\text{height of tree}}{82}$

$\frac{82 \times 5.5}{6.5}$ = height of tree

69 feet = height of tree

TO NEAREST FOOT

Note: Older children could perhaps do these calculations or a handheld calculator could be used. Or, the children could do the measuring and the teacher could do the calculations.